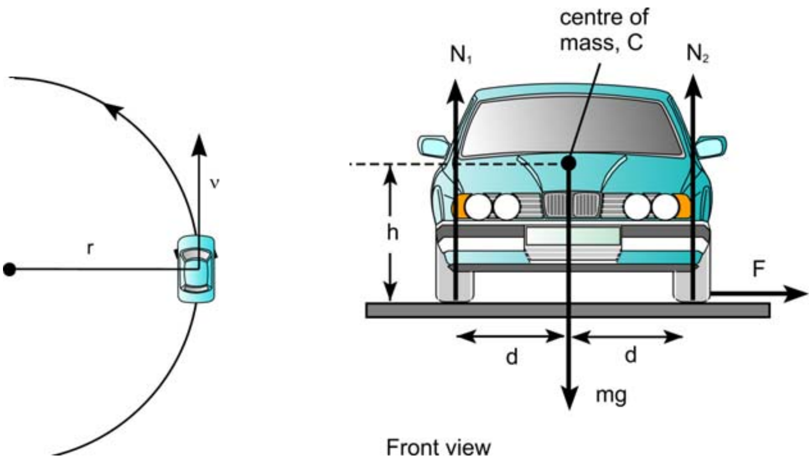
HKDSE PHYSICS

Mechanics : Circular Motion



Part 2

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Circular Motion - Part 2

■ Example 4: Motion of a cyclist rounding a circular track

Consider a cyclist turning around a circular track with radius r and speed v as shown in Fig. 9 and Fig. 10 below.

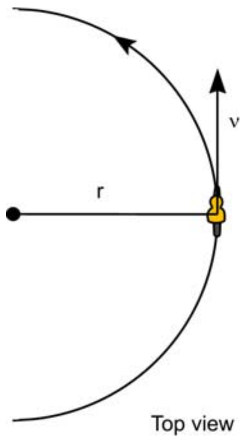

Fig. 9

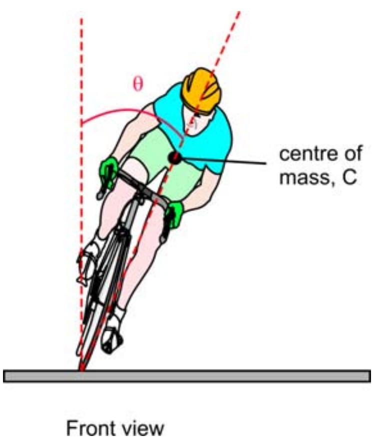

Fig. 10

Let W be the weight of the system (cyclist + bicycle) ($W = mg$)

N = normal force by ground on the system

f = static friction by ground on the system

θ = angle of inclination between cyclist and the vertical

h = height of the centre of mass C of the system above the ground

d = perpendicular distance from the point of contact between the ground and the tyres to the line of application of the weight

Q: Indicate the above 6 quantities in fig.10 above.

Note that the centripetal force required is provided by the **static friction, *f*.**

Referring to Fig. 10:

Horizontally,

$f = m v^2 / r$ (i)

Vertically,

$N = mg$ (ii)

Take moment about the centre of mass (C),

$$N\,d = f\,h \qquad \text{(iii)}$$

Substitute (i) and (ii) into (iii), we have

$$\frac{d}{h} = \frac{v^2}{gr}$$

Since $\tan\theta = d / h$

So $$\tan\theta = \frac{v^2}{gr} \qquad (16)$$

Hence if *v* is large and/or *r* is small, *θ* will have to be increased; otherwise, the bicycle system will topple.

Condition for skidding

Q:

Suppose the coefficient for static friction is μ_s, find the condition for skidding/sliding to occur.

Ans: Condition for skidding is:

Centripetal force, *F*, required > limiting static friction

Since $F = m\,v^2 / r$

and limiting static friction = $\mu_s mg$

So the **maximum speed without skidding is**:

$$v = \sqrt{\mu_s gr} \qquad (17)$$

When $v > \sqrt{\mu_s gr}$, skidding occurs.

■ Example 5: Motion of a car rounding a bend without banking (i.e. on level road)

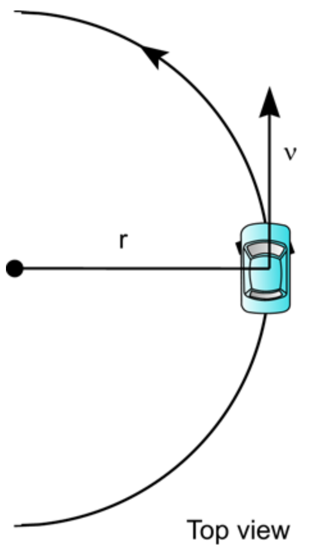

Fig. 11

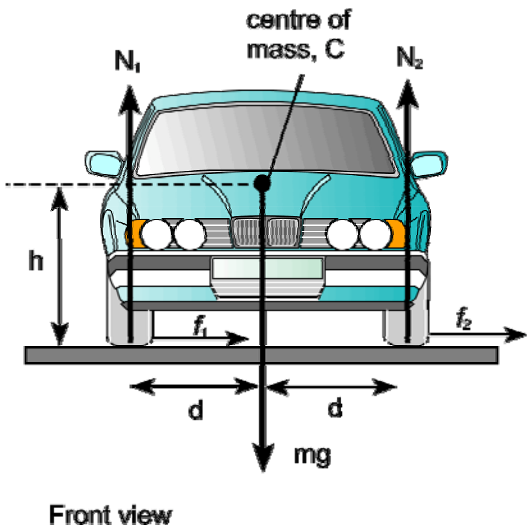

Fig. 12

Fig. 12 is the front view of a car performing circular motion as shown in Fig. 11. f_1 and f_2 are the frictions acting on the right and left wheels respectively. N_1 and N_2 are the normal forces acting on the car. The distance of separation between the right and the left wheels is $2d$.

We want to ask two questions:

i. What is the maximum speed that the car can travel without toppling (overturn)?

ii. If the coefficient for static friction is μ_s ,what is the maximum speed that the car can travel without skidding?

To answer the above two questions, 2 important points must be noted:

1. **The centripetal force required is provided by the static frictions acting on the tyres by the ground.**

2. **For motion without toppling, the net moment of the forces acting on the car about a point must be zero (i.e. total clockwise moment = total anticlockwise moment)**

(i) Maximum speed without toppling

Refer to Fig. 12.
Vertically,

$$N_1 + N_2 = mg \qquad \text{(i)}$$

Take moment about the centre of mass C

$$N_1 d = N_2 d + (f_1 + f_2)\, h$$

$$(N_1 - N_2)\, d = F\, h \qquad \text{(ii)}$$

where F is the centripetal force provided by the frictions

It is clear from (ii) that $N_1 > N_2$ unless F=0. So, as the car turns left, the car's suspension springs are more compressed on the right than on the left, and **so the car tilts outwards**.

Solving (i) and (ii) for N_1 and N_2,

$$N_1 = \frac{mg}{2} + \frac{Fh}{2d}$$

$$N_2 = \frac{mg}{2} - \frac{Fh}{2d}$$

Substitute $F = \frac{mv^2}{r}$ for F, we get,

$$N_1 = \frac{mg}{2} + \frac{mv^2 h}{2rd} \qquad \text{(iii)}$$

$$N_2 = \frac{mg}{2} - \frac{mv^2 h}{2rd} \qquad \text{(iv)}$$

So, as v increases, N_1 increases and N_2 decreases and the car tilts more and more outwards.

The car will overturn when $N_2 < 0$

In view of (iv),

$$\frac{mg}{2} - \frac{mv^2 h}{2rd} < 0$$

$$v > \sqrt{\frac{grd}{h}}$$

The **maximum speed that the car can travel without toppling** is

$$v = \sqrt{\frac{grd}{h}} \qquad (18)$$

(ii) Maximum speed without skidding

The condition for skidding is :

Required centripetal force > Limiting static friction

i.e. $\frac{mv^2}{r} > \mu_s m g$

i.e. $v > \sqrt{\mu_s gr}$

The **maximum speed of the car without skidding** is

$$v = \sqrt{\mu_s gr} \qquad (19)$$

Note the following points:

1. If the speed of the car (v) is too high or the radius of curvature (r) of the path is too small or the road is too wet, the car will skid.
2. Whenever the car skids, it will skid outwards.
3. The **limiting speed** of the car **without skidding** is **independent if its mass**.

Example 6: Motion of a car rounding a bend with banking

Since friction is a limited force, a car rounding a bend may skid if its speed is too fast. If the road is banked at an angle, the **horizontal component of the normal force acting on the car also contributes to the centripetal force, in addition to friction**. The car can then travel at a higher speed without skidding.

Ideal angle of banking

If the road is banked by an angle θ so that the car can round the bend without sideway friction, θ is called the **ideal angle of banking**. In this case, the required centripetal force is provided only by the horizontal component of the normal force *N*, as shown in Fig. 13.

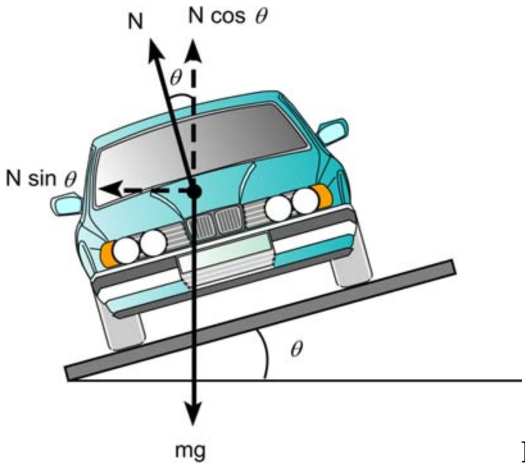

Fig. 13

Refer to Fig. 13.

Vertically,

$$N \cos \theta = mg \qquad \text{(i)}$$

The weight of the car is balanced by the vertical component of the normal force *N*.

Horizontally,

$$N \sin \theta = \frac{mv^2}{r} \qquad \text{(ii)}$$

The horizontal component of the normal force *N* provides the centripetal force.

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